Electromagnetic Current of a Composed Vector Particle in the Light-Front [*]

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Abstract

The violation of the rotational symmetry constraint of the matrix elements of the plus component of the vector current, in the Drell-Yan frame, is studied using an analytical and covariant model of a spin-1 composite particle. The contributions from pair diagrams or zero modes, if missed cause the violation of the rotational symmetry. We reanalyze the prescription suggested by Grach and Kondratyuk [Sov. J. Nucl. Phys. 38, 198 (1984)] to extract the form factors which can eliminate these contributions in particular models.

The light-front description of hadrons [1, 2] in a truncated Fock-space breaks the rotational symmetry, as some rotations are dynamical operators which mixes different components in the Fock-space [3, 4]. The problem to keep the correct rotational properties of a relativistic quantum system, within light-front field theory is difficult to handle, although in principle is solvable, when one is not limited to a Fock-space sector [2]. However, within phenomenological models one is tempted to describe hadrons just with the valence component and calculate observables, in particular the electromagnetic form factors. Thinking on that, one may consider that an analysis with covariant and analytical models, could be useful to give an insight on the properties lost by a description of a composite system in a truncated light-front Fock-space. In that respect, it was studied the rotational symmetry breaking of the plus component of the electromagnetic current ($J^+ = J^0 + J^3$) in the Breit-frame respecting the Drell-Yan condition (purely transverse momentum transfer and $q^+ = q^0 + q^3 = 0$), using an analytical model for the spin-1 vertex of a two-fermion bound state [3]. Following this work, it was pointed out that pair terms give contributions beyond the valence one, and if ignored, the matrix elements of the current break covariance and the angular condition constraint is not fulfilled [4, 5]. Due to that, different prescriptions to extract the form factors from the microscopic matrix elements, which are calculated only with the valence component of the wave function, do not agree [6].

It was found in a numerical calculation of the ρ-meson electromagnetic form factors considering only the valence contribution [3], that the prescription proposed by [6] to evaluate the form-factors, produced results in agreement with the covariant calculations. Later on, Ref. [5] shown that the above prescription eliminates the pair contributions to the form factors, using only a γ^μ structure for the vector meson vertex with the matrix elements of the current taken betweeen light-cone polarization states. Our aim here, is to show that this nice result also extends for the instant form polarization states in the cartesian representation and with more general forms of the vertex, like derivative coupling (in a special form), which extends the previous conclusion [5].

For spin-1 particles, the electromagnetic current has the general form [7]:

$$J^\mu_{\alpha\beta} = [F_1(q^2)g_{\alpha\beta} - F_2(q^2)\frac{q_\alpha q_\beta}{2m_v^2}](p^\mu + p'^\mu) - F_3(q^2)(q_\alpha g^\mu_\beta - q_\beta g^\mu_\alpha) \,, \tag{1}$$

where m_v is the mass of the vector particle, q^μ is the momentum transfer, p^μ and p'^μ is on-shell initial and final momentum respectively. The electromagnetic form factors G_0, G_1 and G_2 are obtained from the covariant form factors F_1, F_2 and F_3 (see also [3]).

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[*] **Talk given in "Light-Cone Workshop: Hadrons and Beyond", Durham, 2003.**

In the impulse approximation, the matrix elements of the electromagnetic current J^+ are written as:

$$J_{ji}^+ = i \int \frac{d^4k}{(2\pi)^4} \frac{Tr[\epsilon_j'^\alpha \Lambda_\alpha'(k, k - p')(k - \not{p}' + m)\gamma^+(k - \not{p} + m)\epsilon_i^\beta \Lambda_\beta(k, k - p)(k + m)]}{((k - p)^2 - m^2 + i\epsilon)((k - p')^2 - m^2 + i\epsilon)(k^2 - m^2 + i\epsilon)} , \qquad (2)$$

where $\gamma^+ = \gamma^0 + \gamma^3$. The polarization four-vectors of the initial and final states are ϵ_i and ϵ_j', respectively. The covariant model for the $^3S_1$ meson vertex has a nonsymmetrical form [3]:

$$\Lambda^\mu(k, k') = \frac{N}{((p - k)^2 - m_R^2 + i\epsilon)^2} \left(\gamma^\mu - \frac{m_v}{2} \frac{k^\mu + k'^\mu}{p.k + m_v m - i\epsilon} \right) , \qquad (3)$$

where N is a normalization factor. $\Lambda'^\mu(k, k') = \Lambda^\mu(k, k')\{p \to p'\}$. The regularization function is used to keep finite Eq. (2). The regularization parameter is m_R.

Here we are going to discuss the pair term contribution to two parts of Eq. (2), namely, the one that has the γ^μ vertex from the initial and final meson and the other in which only the derivative coupling is considered. The remaining terms will be presented in a future work.

Let us begin with the γ^μ structure, for which the trace of Eq. (2) is:

$$Tr[\not{\epsilon}_f^\alpha(k - \not{p}' + m)\gamma^+(k - \not{p} + m)\not{\epsilon}_i^\alpha(k + m)] . \qquad (4)$$

The light-front coordinates are writen as $k^+ = k^0 + k^+$, $k^- = k^0 - k^3$ and $\vec{k}_\perp = (k_x, k_y)$. Next, the terms which contain k^- are separated out, because they may generate nonvanishing Z-diagram contributions even in the limit $q^+ \to 0$ [4]:

$$Tr[\gamma\gamma]_{ji}^{Bad} = \frac{k^-}{2} Tr[\not{\epsilon}_j'^\alpha(k - \not{p}' + m)\gamma^+(k - \not{p} + m)\not{\epsilon}_i^\alpha \gamma^+] , \qquad (5)$$

where "Bad" means the possible contribution of a pair term to the electromagnetic current. The electromagnetic current is computed in the Breit-frame respecting the Drell-Yan condition, therefore the momentum transfer is $q^\mu = (0, q_x, 0, 0)$, $p^\mu = (p^0, -q_x/2, 0, 0)$ for the meson initial state and $p'^\mu = (p^0, q_x/2, 0, 0)$ for the final state. With the definition $\eta = q^2/4m_v^2$, we have $p^0 = m_v\sqrt{1 + \eta}$. The instant-form polarization four-vectors in the cartesian representation for the initial and the final state are given by $\epsilon_x^\mu = (-\sqrt{\eta}, \sqrt{1 + \eta}, 0, 0)$, $\epsilon_x'^\mu = (\sqrt{\eta}, \sqrt{1 + \eta}, 0, 0)$, $\epsilon_y^\mu = \epsilon_y'^\mu = (0, 0, 1, 0)$ and $\epsilon_z^\mu = \epsilon_z'^\mu = (0, 0, 0, 1)$. With these polarization four-vectors, the traces are calculated and the following results are obtained:

$$Tr[\gamma\gamma]_{xx}^{Bad} = k^- \frac{\eta}{8} R \quad , \quad Tr[\gamma\gamma]_{yy}^{Bad} = k^-(k^+ - p^+)^2 ,$$

$$Tr[\gamma\gamma]_{zz}^{Bad} = \frac{1}{8} k^- R \quad , \quad Tr[\gamma\gamma]_{zx}^{Bad} = -k^- \frac{\sqrt{\eta}}{8} R , \qquad (6)$$

where $R = 4 Tr[(k - \not{p}' + m)\gamma^+(k - \not{p} + m)\gamma^-]$.

The integration of the light-front energy, k^-, in Eq. (2) is done using *the pole dislocation method*, developed in Refs. [4, 8], where $q^+ = \delta^+ \to 0_+$. The pair terms or Z-diagram contributions are given by:

$$J_{ji}^{+Z}[\gamma\gamma] = \lim_{\delta^+ \to 0} \int d^4k \frac{\theta(p'^+ - k^+)\theta(k^+ - p^+)Tr[\gamma\gamma]_{ji}^{Bad}}{[1][2][3][4][5]} , \qquad (7)$$

where the square brackets are: $[1] = (k^2 - m^2 + i\epsilon)$; $[2] = ((p - k)^2 - m^2 + i\epsilon)$; $[3] = [2]\{p \to p'\}$; $[4] = ((p - k)^2 - m_R^2 + i\epsilon)^2$ and $[5] = [4]\{p \to p'\}$.

In Eq. (7), we have already isolated the region of $p^+ < k^+ < p'^+$ with $p'^+ = p^+ + \delta^+$, where the pair term contribution to the plus component of the electromagnetic current appears [4]. We have to

consider that the contribution of terms of the form $(k^-)^{m+1}(p^+ - k^+)^n$ in Eq. (7) tends to zero in the limit $\delta \to 0_+$ if $m < n$ [4]. Therefore, one easily gets that $J_{yy}^{+Z}[\gamma\gamma] = 0$.

The other part of Eq. (2) which we analyze here, is the product of the two contributions from the derivative coupling (d) of the vertex, i.e., the second term in Eq. (3). The terms that brought potential contributions to the pair production mechanism in the Drell-Yan frame, are the ones in which k^- appears. These terms written for each cartesian polarization of the initial and final meson are given by:

$$Tr[dd]_{xx}^{Bad} = -(k^-)^3 \frac{\eta}{2} A - (k^{-2}\eta + k^- q_x \sqrt{\eta}\sqrt{1+\eta})B \,,$$

$$Tr[dd]_{zx}^{Bad} = (k^-)^3 \frac{\sqrt{\eta}}{2} A + [k^{-2}\sqrt{\eta} - k^- k^+ \sqrt{\eta}(2k_x + \frac{q_x}{2}\sqrt{1+\eta})]B \,,$$

$$Tr[dd]_{zz}^{Bad} = \frac{(k^-)^3}{2} A + (k^{-2} - k^- k^+)B \,, \qquad (8)$$

where

$$A = Tr[(k - p\!\!\!/' + m)\gamma^+(k - p\!\!\!/ + m)\gamma^+] \,,$$

$$B = Tr[(k - p\!\!\!/' + m)\gamma^+(k - p\!\!\!/ + m)(\frac{\gamma^- k^+}{2} - \vec{\gamma}_\perp \cdot \vec{k}_\perp + m)] \,. \qquad (9)$$

The trace $Tr[dd]_{yy}^{Bad}$ vanishes.

As we have discussed above we use the *"pole dislocation method"* [4, 8] to integrate in the region $p^+ < k^+ < p'^+$ with $p'^+ = p^+ + \delta^+$ from which could arises Z-diagram contributions in the limit of $\delta^+ \to 0$:

$$J_{ji}^{+Z}[dd] = \lim_{\delta^+ \to 0} \int d^4k \frac{\theta(p'^+ - k^+)\theta(k^+ - p^+)\,Tr[dd]_{ji}^{+Bad}}{[1][2][3][4][5][6][7]} \frac{m_v^2}{4} \,, \qquad (10)$$

where $[6] = (p^\mu k_\mu + m_q m_v - \imath\epsilon)$ and $[7] = (p'^\mu k_\mu + m_q m_v - \imath\epsilon)$. One trivially gets that the integrals in Eq. (10) vanishes in the limit of $\delta^+ \to 0$, due to the presence of the denominators $[6]$ and $[7]$.

We remind the reader that the valence contribution to Eq.(2) appears in the interval $0 < k^+ < p^+$, which results from the residue of the pole in the for k^- for $k^2 = m^2$.

In a covariant calculation of the electromagnetic current of a vector particle, the matrix elements of J^+ in the Breit-frame with $q^+ = 0$, satisfy the angular condition equation [6]:

$$\Delta(q^2) = (1 + 2\eta)I_{11}^+ + I_{1-1}^+ - \sqrt{8\eta}I_{10}^+ - I_{00}^+ = 0 \,, \qquad (11)$$

where the matrix elements in the light-cone polarization states are denoted by $I_{m'm}^+$. For the instant form spin basis, the angular condition is given by $J_{yy}^+ = J_{zz}^+$. For light-front models there are several possible forms, or prescriptions, to combine the four independent matrix elements and extract the three electromagnetic form factors, which are nonequivalent if the angular condition is violated. The prescription suggested by Grach and Kondratyuk [6], eliminates the matrix element I_{00}^+, using the angular condition Eq. (11), and the form factors are written as:

$$G_0^{GK} = \frac{1}{3}[J_{xx}^+ + J_{yy}^+ (2 - \eta) + \eta J_{zz}^+] \,, \quad G_1^{GK} = [J_{yy}^+ - J_{zz}^+ - \frac{J_{zx}^+}{\eta}] \,,$$

$$G_2^{GK} = \frac{\sqrt{2}}{3}[J_{xx}^+ + J_{yy}^+ (-1 - \eta) + \eta J_{zz}^+] \,, \qquad (12)$$

where the transformation of the light-cone to the instant form polarization states were performed [3].

Using Eqs. (6) and (7) one derives the following identities:

$$J_{xx}^{+Z}[\gamma\gamma] = -\eta \, J_{zz}^{+Z}[\gamma\gamma] \quad \text{and} \quad J_{zx}^{+Z}[\gamma\gamma] = -\sqrt{\eta} \, J_{zz}^{+Z}[\gamma\gamma] \,, \qquad (13)$$

which by substitution in Eq. (12) implies that the contribution of the pair terms to the form factors computed with that prescription vanishes:

$$
\begin{aligned}
G_0^{GK,Z} &= \frac{1}{3}[J_{xx}^{+Z}[\gamma\gamma] + \eta J_{zz}^{+Z}[\gamma\gamma]] = 0 \,, \quad G_1^{GK,Z} = [-J_{zz}^{+Z}[\gamma\gamma] - \frac{J_{zx}^{+Z}[\gamma\gamma]}{\eta}] = 0 \,, \\
G_2^{GK,Z} &= \frac{\sqrt{2}}{3}[J_{xx}^{+Z}[\gamma\gamma] + \eta J_{zz}^{+Z}[\gamma\gamma]] = 0 \,.
\end{aligned}
\tag{14}
$$

We remind that $J_{yy}^{+Z}[\gamma\gamma] = 0$ in the above equations.

The computed current from the derivative vertex, Eq. (10), does not have Z-diagram terms. Therefore, the form factors for a model based solely on this vertex piece, are independent on the prescription used. To complete the study of the vector particle model defined by Eq. (3), the terms that contain the product of the γ^μ coupling and derivative, should be included in the computation of the form factors.

In summary, using the *pole dislocation method*, we have computed the contribution of the Z-diagram to the plus component of the current, of a composite vector particle, in the Breit-frame constrained by the Drell-Yan condition. In the case of the γ^μ vertex form, we have shown that the cancellation of the Z-diagram contribution for a particular prescription is verified, using matrix elements evaluated in the instant form polarization spin basis, which generalizes a previous work [5]. Although, we have not used all the structure of the $^3S_1$ vector meson vertex to evaluate the current, we have pointed out that the derivative form of the coupling between the quarks and the meson, when weighted by a adequate function does not produce a Z-diagram. The full calculation of the Z-diagram for the vector meson vertex shown in this work is in progress, as well the use of a symmetrical vertex (see e.g. [9]). One could apply the results of such analysis to realistic studies for the ρ-meson or deuteron elastic photo-absorption processes.

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